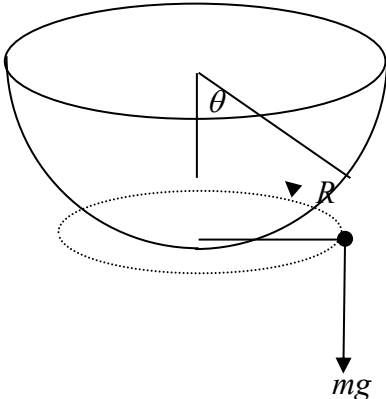
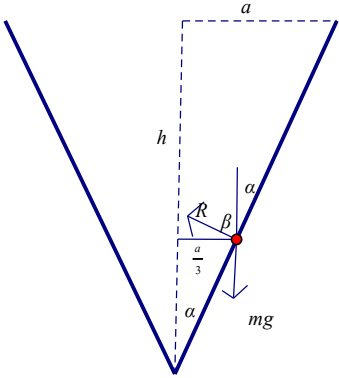


AS and A level Further Mathematics Practice Paper – Horizontal circles
 Mark scheme

Question	Scheme	Marks
1	<div style="text-align: center;">  </div> $R \sin \theta = m \times 4r \sin \theta \times \frac{3g}{8r}$ $R = \frac{3}{2} mg$ $R \cos \theta = mg$ $\frac{3}{2} mg \cos \theta = mg$ $\cos \theta = \frac{2}{3}$ $OC = 4r \cos \theta = 4r \times \frac{2}{3} = \frac{8}{3} r \text{ oe}$	<p>M1A1A1</p> <p>M1A1</p> <p>M1(dep)</p> <p>A1</p> <p>M1A1</p>
		(9 marks)

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
2	 <p>Vertical: $R \cos \beta = mg$</p> <p>Horizontal: $R \sin \beta = \frac{mv^2}{r} = \frac{3mv^2}{a}$</p> <p>Divide: $\tan \beta = \frac{3mv^2}{amg}$</p> $\tan \beta = \frac{h}{a}$ $\frac{3mv^2}{amg} = \frac{h}{a}, \quad \frac{3v^2}{g} = h, \quad v = \sqrt{\frac{hg}{3}} \quad *AG*$	<p>M1A1</p> <p>M1A1</p> <p>M1dep</p> <p>B1</p> <p>A1</p>
		(7 marks)
3	<p>$R(\uparrow) \quad R = mg$</p> <p>$F = \mu mg$</p> <p>$20 \text{ revs per min} = \frac{20}{60} \times 2\pi \text{ rad s}^{-1}$</p> $\left(= \frac{2}{3} \pi \text{ rad s}^{-1} \right)$ <p>$R(\rightarrow) \quad \mu mg = m \times 0.4 \times \left(\frac{2}{3} \pi \right)^2$</p> $\mu = \frac{0.4 \times 4\pi^2}{9g}$ <p>$\mu = 0.18 \text{ or } 0.179$</p>	<p>B1</p> <p>M1A1</p> <p>M1A1ft</p> <p>A1</p>
		(6 marks)

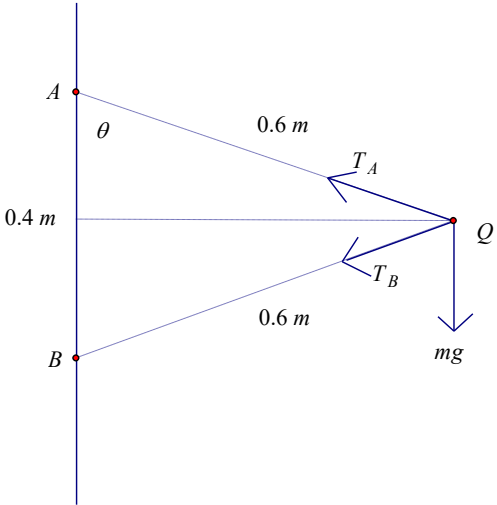
AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
4	$1.2mg \cos \theta = mg$ or $T \cos \theta = mg$ $\cos \theta = \frac{1}{1.2}$ $\theta = \cos^{-1} \frac{1}{1.2}$, $\theta = 33.55\dots$ (accept 34, 33.6 or better) $1.2mg \sin \theta = mr\omega^2$ or $T \sin \theta = mr\omega^2$ $1.2mg \sin \theta = m \times l \sin \theta \omega^2$ $1.2mg = 58.8lm \Rightarrow l = \frac{1.2 \times 9.8}{58.8} = 0.2(\text{m})$	M1A1 A1 M1A1 A1 dM1A1
		(8 marks)
5(a)	$T \sin 60^\circ + R \sin 60^\circ = mg$ $T \cos 60^\circ - R \cos 60^\circ = ml \cos 60^\circ \omega^2$ $T = \frac{1}{2}m(l\omega^2 + \frac{2}{\sqrt{3}}g)$	M1 A1 M1 A1 A1 DM1 A1 (7)
5(b)	$R = \frac{1}{2}m(\frac{2}{\sqrt{3}}g - l\omega^2)$ $\frac{1}{2}m(\frac{2}{\sqrt{3}}g - l\omega^2) > 0$ $\omega < \sqrt{\frac{2g}{l\sqrt{3}}}$ $t > 2\pi \sqrt{\frac{l\sqrt{3}}{2g}}$ **	M1 A1 DM1 A1 DM1 A1 (6)
		(13 marks)

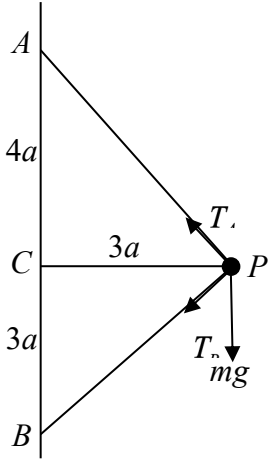
AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
6	<p>For Q $T = 2mg$</p> <p>For P $T \cos \theta = mg$</p> <p style="padding-left: 40px;">$\cos \theta = \frac{1}{2} \quad \theta = 60^\circ \quad *$</p> <p>For $P \rightarrow T \sin \theta = mr\omega^2$</p> <p style="padding-left: 40px;">$2mg \sin \theta = m \times 5l \sin \theta \times \omega^2$</p> <p style="padding-left: 40px;">$\omega^2 = \frac{2g}{5l} \quad \omega = \sqrt{\frac{2g}{5l}} \quad *$</p>	<p>B1</p> <p>M1</p> <p>A1cso</p> <p>M1A1</p> <p>M1depA1</p> <p>A1cso</p>
		(8 marks)

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
7	 <p> $\cos \theta = \frac{0.2}{0.6} \left(= \frac{1}{3} \right)$ </p> <p>Resolve vertically:</p> $T_A \cos \theta = T_B \cos \theta + mg \quad (T_A = T_B + 3mg)$ <p>Acceleration towards the centre:</p> $T_A \sin \theta + T_B \sin \theta = m \times 0.6 \sin \theta \times \omega^2 \quad \left(T_A + T_B = 5 \times \frac{3}{5} \times 100 = 300 \right)$ <p>Substitute values for \square and trig functions and solve to find T_A or T_B</p> $T_B + 147 + T_B = 300, \quad 2T_B = 300 - 147 = 153$ $T_A = 223.5(\text{N}) \quad , \quad T_B = 76.5(\text{N})$ <p> $T_A = 224$ or 220 $T_B = 76$ $T_B = 76.5$ or 77 $T_A = 223$ </p>	<p>B1</p> <p>M1 A2,1,0</p> <p>M1 A2,1,0</p> <p>M1</p> <p>A1 A1</p> <p>(10 marks)</p>

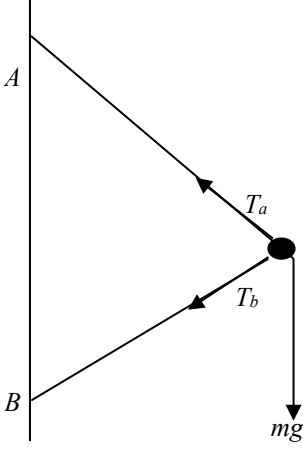
AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
<p>8(a)</p>	 <p> $\cos \theta = \frac{4}{5}$ or $\sin \theta = \frac{3}{5}$ </p> <p> $R(\text{vert}) \quad T_B \cos 45 + mg = T_A \cos \theta$ </p> $\frac{1}{\sqrt{2}} T_B + mg = \frac{4}{5} T_A$ <p> $R(\text{horiz}) \quad T_A \sin \theta + T_B \cos 45 = m \times 3a\omega^2$ </p> $\frac{3}{5} T_A + \frac{1}{\sqrt{2}} T_B = 3ma\omega^2$ $\frac{3}{5} T_A - mg = 3ma\omega^2 - \frac{4}{5} T_A$ $\frac{7}{5} T_A = 3ma\omega^2 + mg$ $T_A = \frac{5}{7} m(3a\omega^2 + g) \quad *$	<p>B1</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>M1</p> <p>A1</p> <p>(8)</p>
<p>8(b)</p>	$T_b = \sqrt{2} \left(\frac{4}{5} T_A - mg \right)$ $= \sqrt{2} \left(\frac{4}{7} m(3a\omega^2 + g) - mg \right)$ $= \frac{3\sqrt{2}}{7} m(4a\omega^2 - g) \quad \text{oe}$	<p>M1</p> <p>A1</p> <p>(2)</p>

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
8(c)	$T_b \leq 0 \Rightarrow 4a\omega^2 \leq g$ $\omega^2 \leq \frac{g}{4a}$ $\omega \leq \frac{1}{2}\sqrt{\frac{g}{a}} \quad *$ <p>(Allow strict inequalities in (c).)</p>	<p>M1</p> <p>A1</p> <p>(2)</p>
		(12 marks)

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
9		
9(a)	$r = \frac{l}{\div 2}$ $R(\uparrow) \quad T_a \cos 45 = T_b \cos 45 + mg$ $T_a - T_b = mg \div 2 \quad (1)$ $R(\rightarrow) \quad T_a \cos 45 + T_b \cos 45 = mr\omega^2$ $T_a \times \frac{1}{\div 2} + T_b \times \frac{1}{\div 2} = m \frac{l}{\div 2} \omega^2$ $T_a + T_b = ml\omega^2 \quad (2)$ $T_a - T_b = mg \div 2 \quad (1)$ $2T_a = m(l\omega^2 + g \div 2)$ $T_a = \frac{1}{2}m(l\omega^2 + g \div 2)$ $T_b = ml\omega^2 - T_a$ $= \frac{1}{2}m(l\omega^2 - g \div 2)$	<p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(8)</p>
9(b)	$T_b > 0 \quad \frac{1}{2}m(l\omega^2 - g \div 2) > 0$ $\omega^2 > \frac{g \div 2}{l} \quad *$	<p>M1</p> <p>A1</p> <p>(2)</p>
		(10 marks)

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

Question	Scheme	Marks
<p>10(a)</p>	$T_A \cos 30^\circ = mg + T_B \cos 30^\circ$ $T_A - T_B = \frac{2mg}{\sqrt{3}}$ $\text{Radius} = \frac{1}{2}l \tan 30^\circ \left(= \frac{\sqrt{3}}{6}l \text{ oe} \right)$ $T_A \cos 60^\circ + T_B \cos 60^\circ = mr\omega^2 = m \left(\frac{1}{2}l \tan 30^\circ \right) \omega^2$ $T_A + T_B = \frac{ml\omega^2}{\sqrt{3}}$ $T_A = \frac{1}{2} \left(\frac{2mg}{\sqrt{3}} + \frac{ml\omega^2}{\sqrt{3}} \right) = \frac{m\sqrt{3}}{6} (2g + l\omega^2) \quad *$ $T_B = \frac{1}{2} \left(\frac{ml\omega^2}{\sqrt{3}} - \frac{2mg}{\sqrt{3}} \right) \text{ oe}$	<p>M1 A1</p> <p>B1</p> <p>M1 A1 A1ft</p> <p>DM1 A1 cso</p> <p>A1 cso</p> <p>(9)</p>
<p>10(b)</p>	$T_B > 0 \quad 2mg < ml\omega^2$ $\omega^2 > \frac{2g}{l}$ $T = \frac{2\pi}{\omega} \quad T < 2\pi \sqrt{\frac{l}{2g}} = \pi \sqrt{\frac{2l}{g}} \quad *$	<p>M1</p> <p>A1</p> <p>DM1 A1 cso</p> <p>(4)</p>
		<p>(13 marks)</p>

AS and A level Further Mathematics Practice Paper – Horizontal circles
Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	M3 2014	1	1.1	Motion in a horizontal circle	1.1b, 3.1b
2	M3 2013R	1	1.1	Horizontal circles	1.1b, 3.1b
3	M3 2013	1	1.1	Horizontal circles	1.1b, 3.1b
4	M3 2017	2	1.1	Motion in a horizontal circle	1.1b, 3.1b, 3.4
5	M3 2014R	2	1.1	Motion in a horizontal circle	1.1b, 2.1, 2.2a, 3.1b, 3.4
6	M3 2013	3	1.1	Horizontal circles	1.1b, 3.1b
7	M3 2012	3	1.1	Horizontal circles	1.1b, 3.1b, 3.4
8	M3 2011	4	1.1	Horizontal circles	1.1b, 2.1, 2.2a, 3.1b, 3.4
9	M3 Jan 2011	5	1.1	Horizontal circles	1.1b, 2.2a, 2.5, 3.1b, 3.2a, 3.4
10	M3 2016	5	1.1	Motion in a horizontal circle	1.1b, 2.1, 2.2a, 3.1b, 3.4